Package ‘nor1mix’

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Description One-dimensional Normal Mixture Models Classes, for, e.g.,
density estimation or clustering algorithms research and teaching;
providing the widely used Marron-Wand densities. Efficient random
number generation and graphics; now fitting to data by ML (Maximum
Likelihood) or EM estimation.
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Transform Clustering / Grouping to Normal Mixture

Description

Simple transformation of a clustering or grouping to a normal mixture object (class "norMix", see, `normix`).

Usage

```r
clus2norMix(gr, x, name = deparse(sys.call()))
```

Arguments

- `gr`: a grouping/clustering vector with values in `{1,..,K}`; possibly a `factor`.
- `x`: numeric vector of (original) data (of the same length as `gr`).
- `name`: name for `normix()` object; constructed from the call by default.

Value

A call to `normix()` with `(mu, sig2, w)` set to the empirical values of the groups (as defined by `split(x,gr)`).

Note

Via this function, any simple clustering algorithm (such as `pam`) can be used as simple mixture model fitting procedure.

Author(s)

Martin Maechler, Dec. 2007

See Also

- `normix`; further `pam()` (or `clara()`) from package `cluster` for sensible clusterings.

Examples

```r
x9 <- rnorMix(500, MW.nm9)
require("cluster")
pxc <- pam(x9, k=3)
plot(pxc, which = 2)# silhouette

(nm.p9 <- clus2norMix(pxc$clustering, x9))
plot(nm.p9, p.norm=FALSE)
lines(MW.nm9, col="thistle")
```
Description

Evaluate the density function of the normal mixture specified as `normix` object.

Usage

```r
dnormix(x, obj, log = FALSE)
dnormixL(obj, x = NULL, log = FALSE, xlim = NULL, n = 511)
dpnormix(x, obj, lower.tail = TRUE)
```

Arguments

- `obj`: an object of class `normix`.
- `x`: numeric vector with abscissa values where to evaluate the density (and probability, for `dpnormix`). For `dnormixL()` by default, when `NULL`, it is constructed from `n` (and `xlim` if that is specified).
- `log`: logical indicating log-density values should be returned.
- `xlim`: range of abscissa values, used if `x = NULL`. By default, `xlim` is taken as mean plus/minus 3 standard deviations of the normal mixture.
- `n`: number of abscissa values to generate if `x` is not specified.
- `lower.tail`: logical; if TRUE (default), probabilities are $P[X \leq x]$, otherwise, $P[X > x]$.

Value

- `dnormix(x)` returns the numeric vector of density values $f(x)$, logged if `log` is TRUE.
- `dnormixL()` returns a list with components
  - `x`: the abscissa values.
  - `y`: the density values $f(x)$ as for `dnormix()`.
- `dpnormix()` returns a list with components
  - `d`: the density values $f(x)$ as for `dnormix()`.
  - `p`: the probability values $F(x)$ as for `pnormix()`.

See Also

`rnormix` for random number generation, and `normix` for the construction and further methods, particularly `plot.normix` which makes use `dnormix`. 
Examples

```r
ff <- dnormmix(MW.nm7)
str(ff)
plot(ff, type = "h", ylim = c(0,1)) # rather use plot(ff, ...)

x <- seq(-4.5, length=501)
dp <- dnormmix(x, MW.nm7)
lines(x, dp$d, col = "tomato", lwd=3)
lines(x, dp$p, col = 3, lwd=2)# does not fit y-wise
if (!all.equal(dp$d, dnormmix(x, MW.nm7), tolerance=1e-12),
    all.equal(dp$p, pnormmix(x, MW.nm7), tolerance=1e-12))
```

llnorMix

**Likelihood, Parametrization and EM-Steps For 1D Normal Mixtures**

**Description**

These functions work with an almost unconstrained parametrization of univariate normal mixtures.

llnorMix(p, *) computes the log likelihood,

obj <- par2norMix(p) maps parameter vector p to a norMix object obj,

p <- nM2par(obj) maps from a norMix object obj to parameter vector p,

where p is always a parameter vector in our parametrization.

Partly for didactical reasons, the following functions provide the basic ingredients for the EM algorithm (see also norMixEM) to parameter estimation:

estep.nm(x, obj, p) computes 1 E-step for the data x, given either a "norMix" object obj or parameter vector p.

mstep.nm(x, z) computes 1 M-step for the data x and the probability matrix z.

emstep.nm(x, obj) computes 1 E- and 1 M-step for the data x and the "norMix" object obj.

where again, p is a parameter vector in our parametrization, x is the (univariate) data, and z is a $n \times m$ matrix of (posterior) conditional probabilities, and $\theta$ is the full parameter vector of the mixture model.

**Usage**

```r
llnorMix(p, x, m = (length(p) + 1)/3)
par2norMix(p, name = sprintf("{from %s}", deparse(substitute(p)))[1]))
nM2par(obj)
estep.nm(x, obj, par)
mstep.nm(x, z)
emstep.nm(x, obj)
```
Arguments

- **p, par** numeric vector: our parametrization of a univariate normal mixture, see details.
- **x** numeric: the data for which the likelihood is to be computed.
- **m** integer number of mixture components; this is not to be changed for a given p.
- **name** (for **par2norMix()**): a name for the "norMix" object that is returned.
- **obj** a "norMix" object, see **norMix**.
- **z** a \( n \times m \) matrix of (posterior) conditional probabilities, \( z_{ij} = P(x_i \in C_j|\theta) \), where \( C_j \) denotes the \( j \)-th group ("cluster").

Details

We use a parametrization of a (finite) univariate normal mixture which is particularly apt for likelihood maximization, namely, one whose parameter space is *almost* a full \( \mathbb{R}^m \), \( m = 3k - 1 \).

For a \( k \)-component mixture, we map to and from a parameter vector \( \theta \) (as an \( \mathbb{R} \)-vector) of length \( 3k - 1 \).

For mixture density

\[
\sum_{j=1}^{k} \pi_j \phi((t - \mu_j)/\sigma_j)/\sigma_j,
\]

we logit-transform the \( \pi_j \) (for \( j \geq 2 \)) and log-transform the \( \sigma_j \), such that \( \theta \) is partitioned into

\[
\begin{align*}
\text{p[ 1:(k-1)]:} & \quad p[j] = \text{logit}(\pi_{j+1}) \quad \text{and} \quad \pi_1 \text{ is given implicitly as } \pi_1 = 1 - \sum_{j=2}^{k} \pi_j. \\
\text{p[ k:(2k-1)]:} & \quad p[k-1+ j] = \mu_j, \text{ for } j=1:k. \\
\text{p[2k:(3k-1)]:} & \quad p[2k-1+ j] = \log(\sigma_j), \text{i.e., } \sigma_j^2 = \exp(2 * p[. + j]).
\end{align*}
\]

Value

- **llnorMix()** returns a number, namely the log-likelihood.
- **par2norMix()** returns "norMix" object, see **norMix**.
- **nM2par()** returns the parameter vector \( \theta \) of length \( 3k - 1 \).
- **estep.nm()** returns \( z \), the matrix of (conditional) probabilities.
- **mstep.nm()** returns the model parameters as a list with components \( w, mu, \) and \( sigma \), corresponding to the arguments of **norMix()**. (and see the 'Examples' on using \texttt{do.call(norMix, \*)} with it.)
- **emstep.nm()** returns an updated "norMix" object.

Author(s)

Martin Maechler

See Also

- **norMix**, **logLik**. Note that the log likelihood of a "norMix" object is directly given by \( \text{sum(dnorMix}(x, \text{, obj, log=TRUE}) \).
- To fit, using the EM algorithm, rather use **norMixEM()** than the \texttt{e.step}, \texttt{m.step}, or \texttt{em.step} functions.
- Note that direct likelihood maximization, i.e., MLE, is typically considerably more efficient than the EM, and typically converges well with our parametrization, see **norMixMLE**.
Examples

(obj <- MW.nm10) # "the Claw" -- m = 6 components
length(pp <- nM2par(obj)) # 17 == (3*6) - 1
par2norMix(pp)
## really the same as the initial 'obj' above

## Log likelihood (of very artificial data):
llnorMix(pp, x = seq(-2, 2, length=1000))
set.seed(47)# of more realistic data:
x <- rnorMix(1000, obj)
llnorMix(pp, x)

## Consistency check : nM2par() and par2norMix() are inverses
all.EQ <- function(x,y, tol = 1e-15, ...) all.equal(x,y, tolerance=tol, ...)
stopifnot(all.EQ(llnorMix(x, obj, log=TRUE)),
        llnorMix(pp, x))

## E- and M- steps: ------------------------
rE1 <- estep.nm(x, obj)
rE2 <- estep.nm(x, par=pp) # the same as rE1
z <- rE1
str( rM <- mstep.nm(x, z))
(rEM <- emstep.nm(x, obj))
stopifnot(all.EQ(rE1, rE2),
        all.EQ(rEM, do.call(norMix, c(rM, name=""))))

---

Description

The fifteen density examples used in Marron and Wand (1992)'s simulation study have been used in quite a few subsequent studies, can all be written as normal mixtures and are provided here for convenience and didactical examples of normal mixtures. Number 16 has been added by Jansen et al.

Usage

MW.nm1  # Gaussian
MW.nm2  # Skewed
MW.nm2.old # Skewed(old)
MW.nm3  # Str Skew
MW.nm4  # Kurtotic
MW.nm5  # Outlier
MarronWand

MW.nm6  # Bimodal
MW.nm7  # Separated (bimodal)
MW.nm8  # Asymmetric Bimodal
MW.nm9  # Trimodal
MW.nm10 # Claw
MW.nm11 # Double Claw
MW.nm12 # Asymmetric Claw
MW.nm13 # Asymmetric Double Claw
MW.nm14 # Smooth Comb
MW.nm15 # Discrete Comb
MW.nm16 # Distant Bimodal

Author(s)

Martin Maechler

Source

They have been translated from Steve Marron’s Matlab code, now at http://www.unc.edu/depts/stat-or/miscellaneous/marron/parameters/nmpar.m, however for number 2, the Matlab code had MW.nm2.0ld; and I’ve defined MW.nm2 as from the Annals paper; see also the last example below.

References


For number 16,
Janssen, Marron, Verb... Sarle (1995) ....

Examples

```r
MW.nm10
plot(MW.nm14)

## These are defined as normix() calls in ../R/zMarrWand-dens.R
nms <- ls(pat="^MW[nm]","package:normix")
nms <- nms[order(as.numeric(substring(nms,6)))]
for(n in nms) {
  cat("\n",n,"\n"); print(get(n, "package:normix"))
}

## Plot all of them:
op <- par(mfrow=c(4,4), mgp = c(1.2, 0.5, 0), tcl = -0.2,
  mar = .1 + c(2,2,2,1), oma = c(0,0,3,0))
for(n in nms[-17]) plot(get(n, "package:normix"))
mtext("The Marron-Wand Densities", outer= TRUE, font= 2, cex= 1.6)

## and their Q-Q-plots (not really fast):
prob <- ppoints(N <- 100)
```
for(n in nms[-17])
qqnorm(qnormMix(prob, get(n, "package:normix")), main = n)
mtext("QQ-plots of Marron-Wand Densities", outer = TRUE,
font = 2, cex = 1.6)
par(op)

## "object" overview:
cbind(sapply(nms, function(n) { o <- get(n)
   sprintf("%-18s: K =%2d; rng = [%3.1f, %2.1f]",
     attr(o, "name"), nrow(o),
     min(o[,"mu"] - 3*sqrt(o[,"sig2"])),
     max(o[,"mu"] + 3*sqrt(o[,"sig2"]))
 })
))

## Note that Marron-Wand (1992), p.720 give #2 as
MW.nm2
## the parameters of which at first look quite different from
MW.nm2.old
## which has been the definition in the above "Source" Matlab code.
## It's easy to see that mu_{nm2} = -.3 + 1.2 * mu_{paper},
## and correspondingly, s2_{nm2} = 1.2^2 * s2_{paper}
## such that they are "identical" apart from scale and location:
op. <- par(mfrow=2:1, mgp= c(1.2,.5,0), tcl= -0.2, mar=.1+c(2,2,2,1))
plot(MW.nm2)
plot(MW.nm2.old)
par(op.)

---

### norMix

Mixtures of Univariate Normal Distributions

**Description**

Objects of class norMix represent finite mixtures of (univariate) normal (aka Gaussian) distributions. Methods for construction, printing, plotting, and basic computations are provided.

**Usage**

```r
norMix(mu, sig2 = rep(1,m), sigma = rep(1,m),
       w = NULL, name = NULL, long.name = FALSE)
```

```r
is.norMix(obj)
m.norMix(obj)
var.norMix(x, ...)
## S3 method for class 'norMix'
mean(x, ...)
## S3 method for class 'norMix'
print(x, ...)
## S3 method for class 'norMix'
x[i,j, drop=TRUE]
```
norMix

Arguments

mu numeric vector of length $K$, say, specifying the means $\mu$ of the $K$ normal components.

sig2 deprecated! numeric vector of length $K$, specifying the variances $\sigma^2$ of the $K$ normal components. Do specify sigma instead!

sigma numeric vector of length $K$, specifying the standard deviations $\sigma$ of the $K$ normal components.

w numeric vector of length $K$, specifying the mixture proportions $\pi_j$ of the normal components, $j = 1, \ldots, K$. Defaults to equal proportions

name optional name tag of the result (used for printing).

long.name logical indicating if the name attribute should use punctuation and hence be slightly larger than by default.

obj,x an object of class norMix.

i,j,drop for indexing, see the generic [ extractor function.

... further arguments passed to methods.

Details

The (one dimensional) normal mixtures, $\mathbb{R}$ objects of class "norMix", are constructed by norMix and tested for by is.norMix. m.norMix() returns the number of mixture components; the mean() method for class "norMix" returns the (theoretical / true) mean $E[X]$ and var.norMix() the true variance $E[(X - E[X])^2]$ where $X \sim \langle \text{norm.mixt} \rangle$.

The subsetting aka "extract" method (x[i,j]: for generic [])—when called as x[i, , ]—will typically return a "norMix" object unless matrix indexing selects only one row in which case x[i, , drop=FALSE] will return the normal mixture (of one component only).

For further methods (density, random number generation, fitting, ...), see below.

Value

norMix returns objects of class "norMix" which are currently implemented as 3-column matrix with column names mu, sigma, and w, and further attributes. The user should rarely need to access the underlying structure directly.

Note

For estimation of the parameters of such a normal mixture, we provide a smart parametrization and an efficient implementation of the direct MLE or also the EM algorithm, see norMixMLE() which includes norMixEM().

Author(s)

Martin Maechler
See Also

dnorMix for the density, pnormMix for the cumulative distribution and the quantile function (qnorMix),
and rnorMix for random numbers and plot.norMix, the plot method.

MarronWand has the Marron-Wand densities as normal mixtures.
norMixMLE() and norMixEM() provide fitting of univariate normal mixtures to data.

Examples

ex <- norMix(mu = c(1,2,5))# defaults: sigma = 1, equal proportions ('w')
ex
plot(ex, p.comp = TRUE)# looks like a mixture of only 2; 'p.comp' plots components

## The 2nd Marron-Wand example, see also ?MWnm2
ex2 <- norMix(name = "#2 Skewed",
             mu = c(0, .5, 13/12),
             sigma = c(1, 2/3, 5/9),
             w = c(.2, .2, .6))
m.norMix (ex2)
mean (ex2)
var.norMix(ex2)
(ex23 <- ex2[2:3,]) # (with re-normalized weights)
stopifnot(is.norMix(ex23),
           all.equal(var.norMix(ex2), 719/1088, tol=1e-14),
           all.equal(var.norMix(ex ), 35/9, tol=1e-14),
           all.equal(var.norMix(ex[2:3,], 13/4, tol=1e-14),
           all.equal(var.norMix(ex23),53^2/(12^3*4),tol=1e-14)
)

plot(ex2, log = "y")# maybe "revealing"

---

norMixFit

**EM and MLE Estimation of Univariate Normal Mixtures**

Description

These functions estimate the parameters of a univariate (finite) normal mixture using the EM algo-
rithm or Likelihood Maximimization via optim(..., method = "BFGS").

Usage

norMixEM(x, m, name = NULL, sd.min = 1e-07* diff(range(x))/m,
         maxiter = 100, tol = sqrt(.Machine$double.eps), trace = 1)
norMixMLE(x, m, name = NULL,
         maxiter = 100, tol = sqrt(.Machine$double.eps), trace = 2)
Arguments

- **x**: numeric: the data for which the parameters are to be estimated.
- **m**: integer or factor: If m has length 1 it specifies the number of mixture components, otherwise it is taken to be a vector of initial cluster assignments, see details below.
- **name**: character, passed to `normix`. The default, NULL, uses `match.call()`.
- **sd.min**: number: the minimal value that the normal components’ standard deviations (sd) are allowed to take. A warning is printed if some of the final sd’s are this boundary.
- **maxiter**: integer: maximum number of EM iterations.
- **tol**: numeric: EM iterations stop if relative changes of the log-likelihood are smaller than tol.
- **trace**: integer (or logical) specifying if the iterations should be traced and how much output should be produced. The default, 1 prints a final one line summary, where `trace = 2` produces one line of output per iteration.

Details

Estimation of univariate mixtures can be very sensitive to initialization. By default, `normixEM` and `normixLME` cut the data into `m` groups of approximately equal size. See examples below for other initialization possibilities.

The EM algorithm consists in repeated application of E- and M- steps until convergence. Mainly for didactical reasons, we also provide the functions `estep.nm`, `mstep.nm`, and `emstep.nm`.

The MLE, Maximum Likelihood Estimator, maximizes the likelihood using `optim`, using the same advantageous parametrization as `llnormix`.

Value

An object of class `normix`.

Author(s)

EM: Friedrich Leisch, originally; Martin Maechler vectorized it in `m`, added `trace` etc.
MLE: M.Maechler

Examples

```r
ex <- normix(mu = c(-1,2,5), sig2 = c(1, 0.5, 3))
plot(ex, col="gray", p.norm=FALSE)

x <- rnormMix(100, ex)
lines(density(x))
rug(x)

## EM estimation may fail depending on random sample
ex1 <- normixEM(x, 3, trace=2) #=> warning (sometimes)
ex1
```
plot(ex1)

## initialization by cut() into intervals of equal length:
ex2 <- normixEM(x, cut(x, 3))
ex2

## initialization by kmeans():
k3 <- kmeans(x, 3)$cluster
ex3 <- normixEM(x, k3)
ex3

## Now, MLE instead of EM:
exM <- normixMLE(x, k3, tol = 1e-12, trace=4)
exM

## real data
data(faithful)
plot(density(faithful$waiting, bw = "SJ"), ylim=c(0,0.044))
rug(faithful$waiting)

(nmF <- normixEM(faithful$waiting, 2))
lines(nmF, col=2)
## are three components better?
nmF3 <- normixEM(faithful$waiting, 3, maxiter = 200)
lines(nmF3, col="forestgreen")

plot.normix

Plotting Methods for 'norMix' Objects

Description

The plot and lines methods for norMix objects draw the normal mixture density, optionally additionally with a fitted normal density.

Usage

## S3 method for class 'norMix'
plot(x, type = "l", n = 511, xout = NULL, xlim = NULL, ylim,
     xlab = "x", ylab = "f(x)", main = attr(x, "name"), lwd = 1.4,
     p.norm = !p.comp, p.h0 = TRUE, p.comp = FALSE,
     parNorm = list(col = 2, lty = 2, lwd = 0.4),
     parH0 = list(col = 3, lty = 3, lwd = 0.4),
     parComp = list(col = "blue3", lty = 3, lwd = 0.4), ...)

## S3 method for class 'norMix'
lines(x, type = "l", n = 511, xout = NULL,
      lwd = 1.4, p.norm = FALSE, parNorm = list(col = 2, lty = 2, lwd = 0.4),
      ...)
Arguments

- **x**: object of class `normix`.
- **type**: character denoting type of plot, see, e.g., `lines`.
- **n**: number of points to generate if `xout` is unspecified.
- **xout**: numeric or NULL giving the abscissae at which to draw the density.
- **xlim**: range of x values to use; particularly important if `xout` is not specified where `xlim` is passed to `dnormix` and gets a smart default if unspecified.
- **ylim**: range of y values to use; by default, if not specified (or containing NA), a smart default is used.
- **xlab, ylab**: labels for the x and y axis with defaults.
- **main**: main title of plot, defaulting to the `normix` name.
- **lwd**: line width for plotting with a non-standard default.
- **p.norm**: logical indicating if the normal density with the same mean and variance should be drawn as well.
- **p.h0**: logical indicating if the line \( y = 0 \) should be drawn.
- **p.comp**: logical indicating if the Gaussian components should also be drawn individually.
- **parNorm**: graphical parameters for drawing the normal density if `p.norm` is true.
- **parH0**: graphical parameters for drawing the line \( y = 0 \) if `p.h0` is true.
- **parComp**: graphical parameters for drawing the single components if `p.comp` is true.
- **...**: further arguments passed to and from methods.

Author(s)

Martin Maechler

See Also

`normix` for the construction and further methods, particularly `dnormix` which is used here.

Examples

```r
plot(normix(m=c(0,3), sigma = c(2,1))) # -> var = c(2^2, 1) = c(4, 1)

plot(MW_mm4, p.norm=FALSE, p.comp = TRUE)
plot(MW_mm4, p.norm=FALSE, p.comp = TRUE, ylim = c(0, 2)) # now works
stopifnot(all.equal(c(0,2), par("yaxp")[1:2], tol=1e-15))

# Further examples in ?normix and ?rnormix
```
pnorMix

Normal Mixture Cumulative Distribution and Quantiles

Description

Compute cumulative probabilities or quantiles (the inverse) for a normal mixture specified as \texttt{normix} object.

Usage

\begin{verbatim}
pnorMix(q, obj, lower.tail = TRUE, log.p = FALSE)

qnormix(p, obj, lower.tail = TRUE, log.p = FALSE,
  tol = .Machine$double.eps^0.25, maxiter = 1000, traceRootsearch = 0,
  method = c("interpQspline", "interpspline", "eachRoot", "root2"),
  l.interp = pmax(1, pmin(20, 1000 / m)), n.mu.interp = 100)
\end{verbatim}

Arguments

- \texttt{obj}:
  an object of class \texttt{normix}.
- \texttt{p}:
  numeric vector of probabilities. Note that for all methods but "eachRoot", \texttt{qnormix}(p, \texttt{+}) works with the full vector \texttt{p}, typically using (inverse) interpolation approaches; consequently the result is very slightly dependent on \texttt{p} as a whole.
- \texttt{q}:
  numeric vector of quantiles.
- \texttt{lower.tail}:
  logical; if \texttt{TRUE} (default), probabilities are \( P[X \leq x] \), otherwise, \( P[X > x] \).
- \texttt{log.p}:
  logical; if \texttt{TRUE}, probabilities \texttt{p} are given as \texttt{log(p)}.
- \texttt{tol, maxiter}:
  tolerance and maximal number of iterations for the root search algorithm, see \texttt{method} below and \texttt{uniroot}.
- \texttt{traceRootsearch}:
  logical or integer in \{0, 1, 2, 3\}, determining the amount of information printed during root search.
- \texttt{method}:
  a string specifying which algorithm is used for the “root search”. Originally, the only method was a variation of "eachRoot", which is the default now when only very few quantiles are sought. For large \texttt{m.norMix()}, the default is set to "root2", currently.
- \texttt{l.interp}:
  positive integer for \texttt{method = "interpQspline" or "interpspline"}, determining the number of values in each “mu-interval”.
- \texttt{n.mu.interp}:
  positive integer for \texttt{method = "interpQspline" or "interpspline"}, determining the (maximal) number of mu-values to be used as knots for inverse interpolation.
Details

Whereas the distribution function \( p\text{norMix} \) is the trivial sum of weighted normal probabilities \( (p\text{norm}) \), its inverse, \( q\text{norMix} \) is computed numerically: For each \( p \) we search for \( q \) such that \( p\text{norMix}(\text{obj}, q) = p \), i.e., \( f(q) = 0 \) for \( f(q) := p\text{norMix}(\text{obj}, q) - p \). This is a root finding problem which can be solved by \texttt{uniroot(f, lower, upper, *)}. If \( \text{length}(p) \leq 2 \) or \texttt{method} = "eachRoot", this happens one for one for the sorted \( p \)’s. Otherwise, we start by doing this for the outermost non-trivial \((0 < p < 1)\) values of \( p \).

For \texttt{method} = "interQpspline" or "interpspline", we now compute \( p \leftarrow p\text{norMix}(q, \text{obj}) \) for values \( q \) which are a grid of length \( l\text{.interp} \) in each interval \([q_j, q_{j+1}]\), where \( q_j \) are the “X-extremes” plus a sub sequence of length \( n\text{.muinterp} \) of the ordered \( \mu[j] \)’s. Then, we use \texttt{montone} inverse interpolation \( \texttt{splinefun}(q, p, \text{method}="\text{mono.H.FC}) \) plus a few (maximally \( \text{maxiter} \), typically one!) Newton steps. The default, "interQpspline", additionally logit-transforms the \( p \) values to make the interpolation more linear. This method is faster, particularly for large \( \text{length}(p) \).

Value

a numeric vector of the same length as \( p \) or \( q \), respectively.

Author(s)

Very first version (for length-1 \( p, q \)) by Erik Jørgensen <Erik.Jorgensen@agrsci.dk>.

See Also

\texttt{dnorMix} for the density function.

Examples

\begin{verbatim}
MW.mm3 # the "strange skew" one
plot(MW.mm3)
## now the cumulative :
x <- seq(-4,4, length=1001)
plot(x, pnorm(x, MW.mm3), type="l", col=2)
## and some of its inverse :
pp <- seq(.1, .9, by=.1)
plot(qnorm(pp, MW.mm3), pp)

## The "true" median of a normal mixture:
median.norMix <- function(x) qnorMix(1/2, x)
median.norMix(MW.mm3) ## -2.32
\end{verbatim}
r.normix

Ratio of Normal Mixture to Corresponding Normal

Description
Compute \( r(x) = f(x)/f_0(x) \) where \( f() \) is a normal mixture density and \( f_0 \) the normal density with the same mean and variance as \( f \).

Usage
\[ \text{r.normix}(\text{obj}, \text{x} = \text{NULL}, \text{xlim} = \text{NULL}, \text{n} = 511, \text{xy.return} = \text{TRUE}) \]

Arguments
- \text{obj} an object of class \text{normix}.
- \text{x} numeric vector with abscissa values where to evaluate the density. Default is constructed from \text{n} (and \text{xlim} if specified).
- \text{xlim} range of abscissa values, used if \text{x} == \text{NULL}. By default, \text{xlim} taken as mean plus/minus 3 standard deviations of the normal mixture.
- \text{n} number of abscissa values to generate if \text{x} is not specified.
- \text{xy.return} logical indicating if the result should be a list or just a numeric vector, see below.

Value
It depends on \text{xy.return}. If it’s false, a numeric vector of the same length as \text{x}, if true (as per default), a list that can be plotted, with components
- \text{x} abscissa values corresponding to argument \text{x}.
- \text{y} corresponding values \( r(x) \).
- \text{f0} values of the moment matching normal density \( f_0(x) \).

Note
The ratio function is used in certain semi-parametric density estimation methods (and theory).

Examples
\[ \text{d3} \leftarrow \text{normix}(m = 5*\text{c}(0.2), \text{w} = \text{c}(0.6, 0.3, 0.1)) \]
\[ \text{plot(d3)} \]
\[ \text{rd3} \leftarrow \text{r.normix(d3)} \]
\[ \text{str(rd3)} \]
\[ \text{stopifnot(rd3 \$ y == r.normix(d3, xy.ret = FALSE))} \]
\[ \text{par(new = TRUE)} \]
\[ \text{plot(rd3, type = "l", col = 3, axes = FALSE, xlab = "", ylab="")} \]
\[ \text{axis(4, col.axis=3)} \]
Description

Generate \( n \) random numbers, distributed according to a normal mixture.

Usage

\[
\text{rnormix}(n, \text{obj})
\]

Arguments

- \( n \): the number of random numbers desired.
- \( \text{obj} \): an object of class \text{norMix}.

Details

For a mixture of \( m \), i.e., \( m.\text{norMix}(\text{obj}) \), components, generate the number in each component as multinomial, and then use \text{rnorm} for each.

Value

numeric vector of length \( n \).

See Also

dnorMix for the density, and norMix for the construction and further methods.

Examples

\[
x <- \text{rnormix}(5000, \text{MW.nm10})
\]
\[
\text{hist}(x) \# \text{you don't see the claw}
\]
\[
\text{plot(density}(x), \text{ylim = c(0,0.6)},
\text{main = "Estim. and true 'MW.nm10' density")}
\]
\[
\text{lines(MW.nm10, col = "orange")}
\]
Sort Method for "norMix" Objects

Description
Sorting a "norMix" object (see `normix`), sorts along the \( \mu \) values; i.e., for the default `decreasing = FALSE` the resulting \( x[, "\mu"] \) are sorted from left to right.

Usage
```r
## S3 method for class 'norMix'
sort(x, decreasing = FALSE, ...)
```

Arguments
- **x**: an object of class "norMix".
- **decreasing**: logical indicating if sorting should be up or down.
- **...**: further arguments passed to `sort(x[, "\mu"], *)`.

Value
a "norMix" object like \( x \).

Examples
```r
sort(MW.nm9)
stopifnot(identical(MW.nm2, sort(MW.nm2)))
```
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