This document explains how to use the parameters in an object returned by \texttt{svm()} for own prediction functions.

## 1 Binary Classifier

For class prediction in the binary case, the class of a new data vector $n$ is usually given by the sign of

$$\sum_i a_i y_i K(x_i, n) + \rho$$

where $x_i$ is the $i$-th support vector, $y_i$ the corresponding label, $a_i$ the corresponding coefficient, and $K$ is the kernel (for example the linear one, i.e. $K(u,v) = u^T v$).

Now, the \texttt{libsvm} library interfaced by the \texttt{svm()} function actually returns $a_i y_i$ as $i$-th coefficient and the negative $\rho$, so in fact uses the formula:

$$\sum_i \text{coef}_i K(x_i, n) - \rho$$

where the training examples (=training data) are labeled \{1,-1\} (!). A simplified R function for prediction with linear kernel would be:

```r
svmpred <- function (m, newdata, K=crossprod)
{
  ## this guy does the computation:
  pred.one <- function (x)
    sign(sum(sapply(1:m$tot.nSV, function (j)
        K(m$SV[j,], x) * m$coefs[j]
    )) - m$rho)

  ## this is just for convenience:
  if (is.vector(newdata))
    newdata <- t(as.matrix(x))
  sapply (1:nrow(newdata),
          function (i) pred.one(newdata[i,]))
}
```
where \texttt{pred.one()} does the actual prediction for one new data vector, the remainder is just a convenience for prediction of multiple new examples. It is easy to extend this to other kernels, just replace \texttt{K()} with the appropriate function (see the help page for the formulas used) and supply the additional constants.

As we will see in the next section, the multi-class prediction is more complicated, because the coefficients of the diverse binary SVMs are stored in a compressed format.

2 Multiclass-classifier

To handle \( k \) classes, \( k > 2 \), \texttt{svm()} trains all binary subclassifiers (one-against-one-method) and then uses a voting mechanism to determine the actual class. Now, this means \( k(k-1)/2 \) classifiers, hence in principle \( k(k-1)/2 \) sets of SVs, coefficients and rhos. These are stored in a compressed format:

1. Only one SV is stored in case it were used by several classifiers. The \texttt{model$SV-matrix} is ordered by classes, and you find the starting indices by using \texttt{nSV} (number of SVs):

\[
\texttt{start} \leftarrow \texttt{c(1, cumsum(model$nSV))}
\texttt{start} \leftarrow \texttt{start[-length(start)]}
\]

\texttt{sum(nSV)} equals the total number of (distinct) SVs.

2. The coefficients of the SVs are stored in the \texttt{model$coefs-matrix}, grouped by classes. Because the separating hyperplanes found by the SVM algorithm has SVs on both sides, you will have two sets of coefficients per binary classifier, and e.g., for 3 classes, you could build a block-matrix like this for the classifiers \((i, j) (i, j=\text{class numbers})\):

\[
\begin{array}{ccc}
1 & \backslash & j \\
\hline
0 & 0 & X \\
1 & set (1, 0) & X \\
2 & set (2, 0) & set (2, 1) \\
\end{array}
\]

where \texttt{set(i, j)} are the coefficients for the classifier \((i, j)\), lying on the side of class \( j \). Because there are no entries for \((i, i)\), we can save the diagonal and shift up the lower triangular matrix to get

\[
\begin{array}{ccc}
i & \backslash & j \\
\hline
0 & set (1, 0) & set (0,1) & set (0,2) \\
1 & set (2, 0) & set (2,1) & set (1,2) \\
\end{array}
\]

Each set \((., j)\) has length \texttt{nSV[j]}, so of course, there will be some filling 0s in some sets. \texttt{model$coefs} is the transposed of such a matrix, therefore for a data set with, say, 6 classes, you get 6-1=5 columns.

The coefficients of \((i, j)\) start at \texttt{model$coefs[start[i],j]} and those of \((j, i)\) at \texttt{model$coefs[start[j],i-1]}.

3. The \( k(k-1)/2 \) rhos are just linearly stored in the vector \texttt{model$rho}.
The following code shows how to use this for prediction:

```r
## Linear Kernel function
K <- function(i,j) crossprod(i,j)
predsvm <- function(object, newdata)
{
  ## compute start-index
  start <- c(1, cumsum(object$nSV)+1)
  start <- start[-length(start)]
  
  ## compute kernel values
  kernel <- sapply (1:object$tot.nSV,
                    function (x) K(object$SV[x,], newdata))
  
  ## compute raw prediction for classifier (i,j)
predone <- function (i,j)
  {
    ## ranges for class i and j:
    ri <- start[i] : (start[i] + object$nSV[i] - 1)
    rj <- start[j] : (start[j] + object$nSV[j] - 1)
    
    ## coefs for (i,j):
    coef1 <- object$coefs[ri, j-1]
    coef2 <- object$coefs[rj, i]
    
    ## return raw values:
    crossprod(coef1, kernel[ri]) + crossprod(coef2, kernel[rj])
  }
  
  ## compute votes for all classifiers
  votes <- rep(0,object$nclasses)
  c <- 0 # rho counter
  for (i in 1 : (object$nclasses - 1))
    for (j in (i + 1) : object$nclasses)
      if (predone(i,j) > object$rho[c <- c + 1])
        votes[i] <- votes[i] + 1
      else
        votes[j] <- votes[j] + 1
  
  ## return winner (index with max. votes)
  object$levels[which(votes %in% max(votes))[1]]
}
```

In case data were scaled prior fitting the model (note that this is the default for `svm`), the new data needs to be scaled as well before applying the predition functions, for example using the following code snipped (object is an object returned by `svm`, `newdata` a data frame):

```r
if (any(object$scaled))
  newdata[,object$scaled] <-
    scale(newdata[,object$scaled, drop = FALSE],
          center = object$x.scale$"scaled:center",
          scale = object$x.scale$"scaled:scale"
    )
```

For regression, the response needs to be scaled as well before training, and the predictions need to be scaled back accordingly.